भारतीय प्रौद्योगिकी संस्थान भुवनेश्वर
INDIAN INSTITUTE OF TECHNOLOGY BHUBANESWAR

COURSE DETAILS FOR VARIOUS ACADEMIC PROGRAMME

Name of the Program: Elective for 3rd /4th semester M.Sc. Mathematics students and Ph.D. students of any discipline.

Name of the Schools: proposed by the Department of Mathematics, SBS

1.	**Course Title:**	REPRESENTATION THEORY OF FINITE GROUPS
2.	**Subject Code:**	MA7L033
3.	**L: T: P:**	4-0-0
4.	**Credit:**	4
5.	**Semester No:**	3, 4
6.	**Core/Elective/Breadth:**	Elective
7.	**Contact Hours:**	54 Approx.
8.	**Theory:**	Algebra
9.	**(Autumn/ Spring):**	Both
10.	**Level:**	Second-year (3rd/4th semester) M.Sc. and Ph.D. students.
11.	**Pre- requisite:**	Linear Algebra (MA5L001) and Algebra (MA5L006) or equivalent
12.	**Lab:**	None
13.	**Instructor(s):**	1. Dr. Biplab Paul 2. Dr. Sunil Kumar Prajapati
	DETAILS OF COURSE/SUBJECT ADOPTED FROM:	
14.	**Name of the other IIT/ Universities offering similar subjects:**	See Appendix A: examples of relevant courses at IISC Bangalore, IIT Madras, IIT Delhi and IISER Bhopal are provided.
15.	**Course Name:**	See Appendix A
16.	**Course Code:**	See Appendix A
17.	**L-T-P-C:**	See Appendix A
18.	**Credit:**	See Appendix A

19. **Objective of Course:**

The objective of this course is to provide a comprehensive understanding of how groups act on vector spaces through linear representations. The course aims to develop the theoretical foundations and computational techniques necessary to analyse and classify these representations. Representation theory is quite deep and needed in advanced mathematics (such as number theory, Lie theory, Harmonic Analysis etc) courses as well as Physics and Chemistry.

Learning Outcome: After completing the course, it is intended that the students will be able to:

- Express abstract groups in terms of matrix groups.
- Construct irreducible representations, which are the building blocks to understand abstract groups.
- Able to compute the concrete matrix elements representing corresponding group elements by Schur's Lemma.
- Construct induced representations to get valuable information about some irreducible representations of the parent group.

20. Break-up of lectures and tutorials

S. No.	Topic	No. of Lectures
1	Basic definitions and examples, the tensor product of two representations	5
2	Character Theory: Schur's lemma, orthogonality, decomposition of regular representation.	8
3	Representations of abelian groups, induced representations	5
4	Compact groups and their representations	6
5	Various concrete examples: quaternion group of order 8, dihedral groups, groups of order p^3, symmetric groups, computations with GAP	8
6	Group algebras	5
7	Mackey's criterion, Artin's theorem, Brauer's theorem	17
	Total	**54**

21. Details of Course:

Suggested Syllabus for IIT Bhubaneswar

Definitions, Basic examples, sub-representations, irreducible representations, the tensor product of two representations, symmetric square and alternating square, character of representation, Schur's lemma, orthogonality of characters, character tables of familiar groups: cyclic groups, abelian groups, quaternion group of order 8, dihedral groups, groups of order p^3, symmetric groups and computations with GAP, regular representation, induced representation, representation of compact groups, various examples, group algebra, Mackey's criterion, Artin's theorem, Brauer's theorem.

22. Suggested Texts

- J. P. Serre, Linear Representations of Finite Groups (Graduate Texts in Mathematics), 2nd Edition, Springer-Verlag New York Inc., 1977.
- W. Fulton and J. Harris, Representation Theory: a first course, 2nd Indian reprint, Springer India, 2007.
- B. Steinberg, Representation Theory of Finite Groups: an introductory approach, Springer New York, 2012.

23. Suggested References

- C. W. Curtis and I. Reiner, Representation theory of finite groups and associative algebras, AMS Chelsea Publishing, AMS, 2006.
- G. James and M. Liebeck, Representations and Characters of Groups, Cambridge University Press, 2001.

(Dr. Biplab Paul)   (Dr. Sunil Kumar Prajapati)

Instructor-1 (SBS)   Instructior-2 (SBS)

Course Curriculum Committee (Mathematics, SBS):
1. Chairperson - HOD (Mathematics)

2. Dr. Tarakanta Nayak   3. Prof. V. R. Allu

4. Dr. Bankim Chandra Mandal   5. Dr. Indrajit Jana

HOD (Department of Mathematics)

Head, SBS

Dean (PGRP)

Typical courses offered at other places

1. IISC Bangalore

MA220: Representation Theory of Finite Groups (Credits: 3:0)
Pre-requisites: MA 219 – Linear Algebra, MA 212 – Algebra I.

Course contents
Representation of finite groups, irreducible representations, complete reducibility, Schur's lemma, characters, orthogonality, class functions, regular representations and induced representation, the group algebra.
Linear groups: Representation of the group SU(2).

References

- J. P. Serre, *Linear representations of finite groups*, Graduate Texts in Mathematics. Vol. 42. Springer-Verlag. New York-Heidelberg. 1977.
- Etingof Pavel, Golberg Oleg, Hensel Sebastian, Liu Tiankai, Schwendner Alex, Vaintrob Dmitry, Yudovina Elena, *Introduction to representation theory. With historical interludes by Slava Gerovitch*, Student Mathematical Library 59. American Mathematical Society. 2011.

2. IISER Bhopal

MTH 410/514/620: Representation Theory (Credits: 4:0)
Pre-requisites: MTH 301 - Group Theory, MTH 302 - Rings and Modules.

Course contents
- Representations of groups, subrepresentations, Irreducible representations, tensor product of representations, Maschke's theorem, Wedderburn decomposition
- Characters of representations, Generalized characters, Schurs lemma, Orthogonality, Regular representations, Decomposition theorems
- Representations of direct product of finite groups, Induced representations, Reciprocity theorem
- Representations and characters of standard finite and infinite groups: cyclic groups, dihedral groups, symmetric and alternating groups of small order etc.
- Applications of Representation Theory

References
- J. P. Serre, Linear Representations of Finite groups (Graduate Texts in Mathematics), 2nd Edition, Springer-Verlag New York Inc., 1977.
- W. Fulton and J. Harris, Representation Theory, A First Course, 2nd Indian reprint, Springer India, 2007.
- G. James, M. Liebeck, Representations and Characters of Groups, Cambridge University Press, 2001.
- M. Suzuki, Group Theory II, Springer-Verlag, 1983.

3. IIT Madras

Representation Theory (Code: MA5017 | Credit: 9)
Prerequisite: MA5310-Linear Algebra, MA5320-Algebra-1

Course Contents
Representation of finite groups: Definitions, Basic examples, sub-representations, irreducible representations, the tensor product of two representations, character of representation, Schur's lemma, orthogonality of characters, regular representation, induced representation, representation of compact groups, various examples.

References
- J-P. Serre, Introduction to representation theory, Springer GTM-42.
- W. Fulton and J. Harris, Representation Theory, Springer Verlag, GTM-129.

4. IIT Delhi

Representation of finite groups (Code: MTL739 | L-T-P-C: 3-0-0-3)

Course contents
Revision of basic group theory. Definition and examples of representation. Subrepresentation, sum and tensor product of representations, irreducible representations; Character Theory: Character of a representation, Schur's Lemma, Maschke's theorem, Orthogonality relations for characters, decomposition of regular representation, number of irreducible representations of a group; Representation of subgroups and product of groups, induced representations; Group Algebra: Representations and modules; Decomposition of complex algebra C[G] and Integrability properties of characters. Induced representations, restriction to subgroups, Reciprocity formula, Mackey's irreducibility criterion; Irreducible representations of symmertric groups (S_n) and alternating groups (A_n).