



भारतीय प्रौद्योगिकी संस्थान भुवनेश्वर  
INDIAN INSTITUTE OF TECHNOLOGY BHUBANESWAR

**COURSE DETAILS FOR VARIOUS ACADEMIC PROGRAMME**

**Name of the Program:** Elective-applicable for Mathematics and Computer Science students.

**Elective for 3rd/4<sup>th</sup> semester M.Sc. Mathematics students and Ph.D. Students of any discipline.**

**Name of the Schools:** proposed by SBS

|                                                |                                                                    |                                                                                                                                                  |
|------------------------------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.                                             | Course Title:                                                      | <b>COMMUTATIVE ALGEBRA</b>                                                                                                                       |
| 2.                                             | Subject Code:                                                      | MA7LXXX                                                                                                                                          |
| 3.                                             | L: T: P:                                                           | 3-1-0                                                                                                                                            |
| 4.                                             | Credit:                                                            | 4                                                                                                                                                |
| 5.                                             | Semester No:                                                       | 3, 4                                                                                                                                             |
| 6.                                             | Core/Elective/Breadth:                                             | ELECTIVE                                                                                                                                         |
| 7.                                             | Contact Hours:                                                     | 52 APPROX                                                                                                                                        |
| 8.                                             | Theory:                                                            | ALGEBRA                                                                                                                                          |
| 9.                                             | (Autumn/ Spring):                                                  | BOTH                                                                                                                                             |
| 10.                                            | Level:                                                             | Second year (3 <sup>rd</sup> /4 <sup>th</sup> semester) M.Sc. and Ph.D. students.                                                                |
| 11.                                            | Pre- requisite:                                                    | ALGEBRA (MA5L006)                                                                                                                                |
| 12.                                            | Lab:                                                               | NONE                                                                                                                                             |
| 13.                                            | Instructor(s):                                                     | 1. Dr. Arpan Dutta                                                                                                                               |
| <b>DETAILS OF COURSE/SUBJECT ADOPTED FROM:</b> |                                                                    |                                                                                                                                                  |
| 14.                                            | Name of the other IIT/ Universities offering the similar subjects: | See Appendix: examples of relevant courses at Harvard, IISC Bangalore, IIT Madras, ISI Kolkata, IIT Bombay, IISER Mohali and NISER are provided. |
| 15.                                            | Course Name:                                                       | See Appendix A                                                                                                                                   |
| 16.                                            | Course Code:                                                       | See Appendix A                                                                                                                                   |
| 17.                                            | L-T-P-:                                                            | See Appendix A                                                                                                                                   |
| 18.                                            | Credit:                                                            | See Appendix A                                                                                                                                   |

## 19.Objective of Course:

Commutative Algebra arose during the development of the theory of algebraic numbers and algebraic geometry. From 19<sup>th</sup> century onwards, these two seemingly different topics started showing remarkable analogies. With time, the attempts to solve the problems they posed led to the isolation of a number of general ideas, a thorough treatment of which led to significant developments in the study of commutative rings. In modern mathematics, commutative algebra is one of the foundation stones of algebraic geometry and algebraic number theory. It provides the complete local tools to study the global problems in these areas.

In this course, we aim to provide the student a gentle introduction to the subject. We will start by studying algebraic structures, and then introduce the student to module theory. Afterwards, we will study the concepts of localization, primary decomposition, and integral dependence. Finally, we will cover chain conditions and Noetherian and Artinian rings. Time permitting, we will also introduce the student to dimension theory.

## 20. Break-up of lectures and tutorials

| S.NO. | TOPIC                                               | NO. OF LECTURES | TUTORIALS |
|-------|-----------------------------------------------------|-----------------|-----------|
| 1.    | Introduction to algebraic structures                | 2               | 1         |
| 2.    | Extension and contraction of ideals                 | 1               | 0         |
| 3.    | Modules and module homomorphisms                    | 1               | 1         |
| 4.    | Submodules and Quotient modules                     | 2               | 1         |
| 5.    | Operations on submodules, Direct sum and product    | 2               | 1         |
| 6.    | Finitely generated modules                          | 1               | 0         |
| 7.    | Exact sequences                                     | 1               | 0         |
| 8.    | Tensor products                                     | 1               | 1         |
| 9.    | Restriction and Extension of scalars                | 1               | 0         |
| 10.   | Algebras and Local properties                       | 1               | 0         |
| 11.   | Extended and contracted ideals in ring of fractions | 1               | 1         |
| 12.   | Primary Decomposition                               | 4               | 2         |
| 13.   | Integral dependence                                 | 1               | 1         |
| 14.   | Going-up theorem                                    | 1               | 0         |
| 15.   | Integrally closed domains                           | 2               | 1         |
| 16.   | Going-down theorem                                  | 1               | 1         |
| 17.   | Valuation rings                                     | 2               | 1         |

|       |                                           |    |    |
|-------|-------------------------------------------|----|----|
| 18.   | Chain Conditions                          | 3  | 1  |
| 19.   | Noetherian rings                          | 4  | 2  |
| 20.   | Primary decomposition in Noetherian rings | 1  | 1  |
| 21.   | Artinian Rings                            | 2  | 1  |
| TOTAL |                                           | 35 | 17 |

## 21. Details of Course:

| Suggested Syllabus for IIT Bhubaneswar                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Introduction to algebraic structures, Extension and contraction of ideals; Modules and module homomorphisms, Submodules and Quotient modules, Operations on submodules, Direct sum and product, Finitely generated modules; Exact sequences, Tensor products, Restriction and Extension of scalars, Algebras; Local properties, Extended and contracted ideals in ring of fractions; Primary decomposition; Integral dependence, Going-up theorem, Integrally closed domains, Going-down theorem; Valuation rings; Chain conditions; Noetherian rings, Primary decomposition in Noetherian rings, Artinian Rings.</p> |

## 22. Suggested Texts:

- M. F. Atiyah, I. G. MacDonald, Introduction to Commutative Algebra, Addison Wesley Publishing Co., 1969.
- D. S. Dummit, R. M. Foote, Abstract Algebra, Wiley-India Edition, 2013.

## 23. Suggested References:

- O. Zariski and P. Samuel, Commutative Algebra I, II, Van Nostrand, Princeton (1958, 1960).
- N. Jacobson, Basic Algebra, Vol II, Dover Publications, 2009.
- H. Matsumura, Commutative Algebra, Second Edition, Mathematics lecture notes series, Vol. 56, Benjamin/Cummings, 1980.